

Droplets

A novel marker design for proportional relations

Stefan Lengauer, Peter Waldert, Benedikt Kantz, and Tobias Schreck

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Visualization and User Experience Seminar (VUES) on Spatial Biology



Motivation

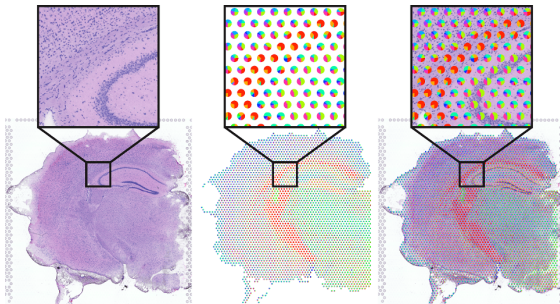


Figure: Taken from http://biovis.net/2024/biovisChallenges_vis/, generated with *STdeconvolve* by Miller et al. Miller, Huang, Atta, et al. [1].

“[...] the only worse design than a pie chart is several of them, for then the viewer is asked to compare quantities located in spatial disarray both within and between pies [...]” – Tufte and Graves-Morris [2]

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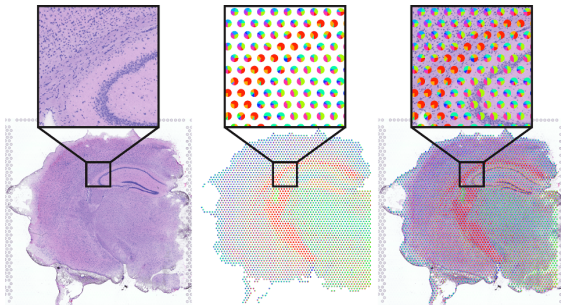


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Design Considerations

- Goals for a marker design
 1. Optimal usage of the available space (as translucent as possible)
 2. Visually enhance coherent regions with similar cell-type compositions
 3. Visually reflect the association to a local cluster

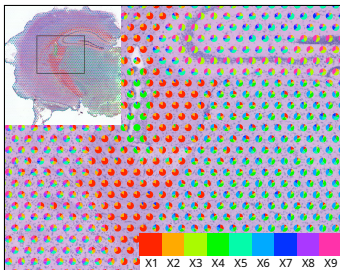
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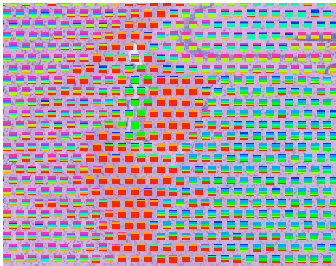
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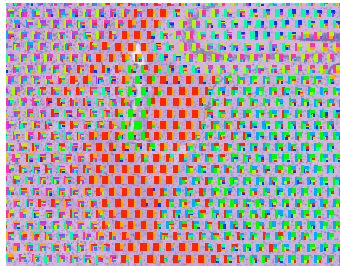
Alternative Designs



(a) Reference: Pie Charts



(b) Bar Charts



(c) Treemaps

Figure: Alternative baseline visualizations for proportional relations: (b) Bar Charts, (c) Treemaps

Alternative Designs

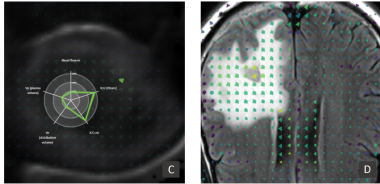


Figure: *Stixels* by Mörrh, Haldorsen, Bruckner, *et al.* [3]

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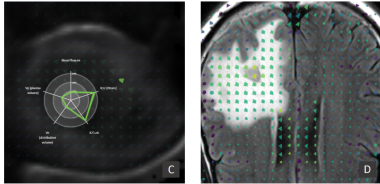


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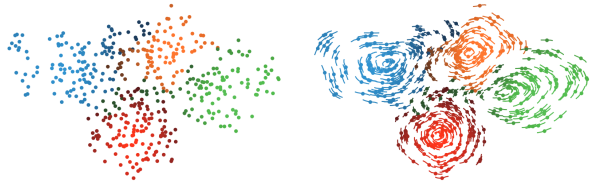


Figure: *Winglets* by Lu, Wang, Lanir, *et al.* [4]

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- Proposed idea
 - Separated, but tightly packed shapes
 - Circular base shape + protruding tail pointing towards the local cluster

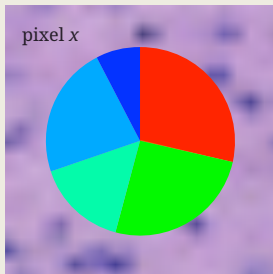
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Terminology

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$x \in X \dots$ The index set of ST pixels

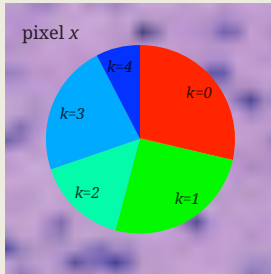
$k \in K \dots$ The index set of classes (cell-types)

$P(x) \in \mathbb{R}^2 \dots$ The pixel x 's position

$w_x^k \dots$ The class k 's contribution to pixel x

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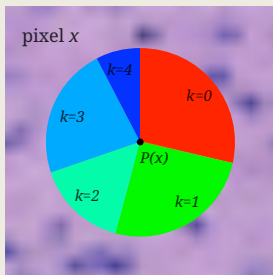
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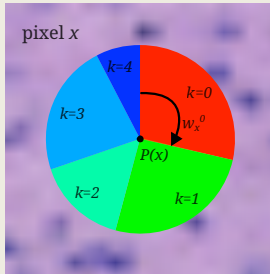
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Droplet Marker

A droplet marker consists of a circular base shape, equipped with a protruding tail.

Droplet definition

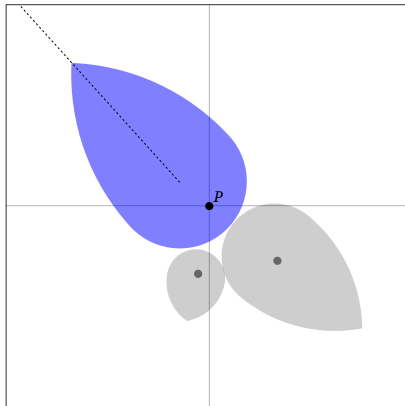
Droplet $D_x^k = \langle r_x^k, \varepsilon_x^k, O_x^k, P_x^k \rangle$ of class k for pixel x with:

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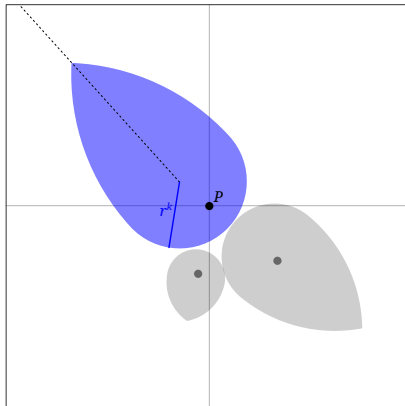
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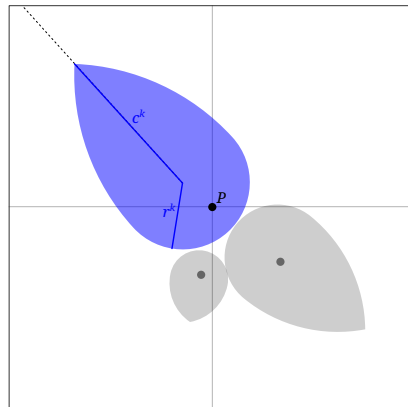
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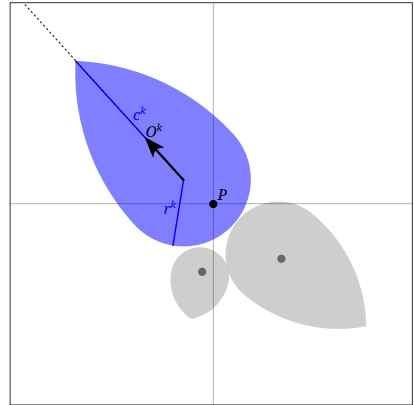
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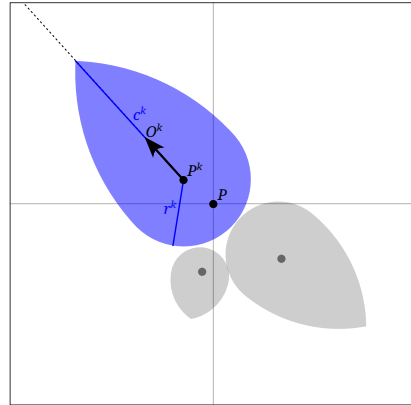
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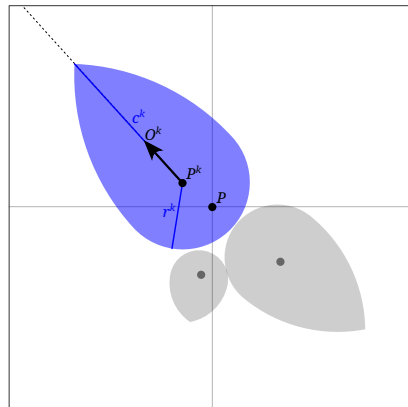
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ε_x^k	... The excentricity	?
O_x^k	... The orientation	?
P_x^k	... The center point	?



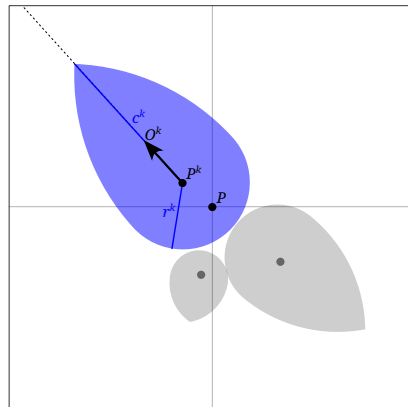
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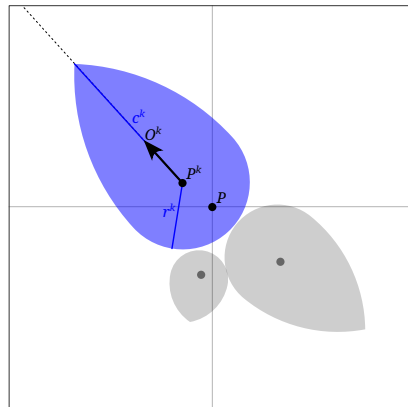
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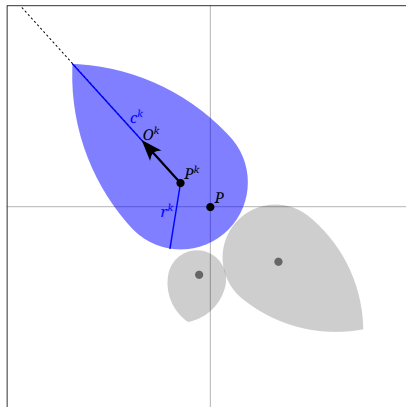
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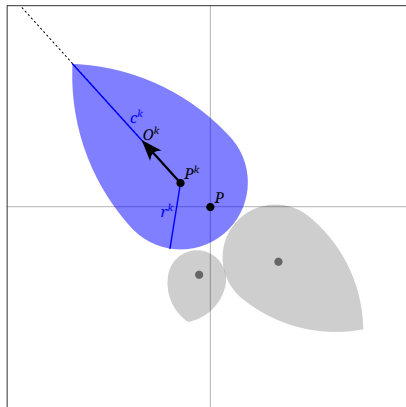
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Attraction Potential

A droplet is attracted by all other droplets of same class, relative to their respective contributions w_x^k and their distances.

Scalar field Φ^k

Attraction potential for class k at $p \in \mathbb{R}^2$:

$$\Phi^k(p) = \sum_{x \in X} \omega_x^k \theta(\|P(x) - p\|)$$

$$\Phi_x^k := \Phi^k(P(x))$$

$\omega_x^k \propto w_x^k \dots$ Weight

$\theta(r) = e^{-\left(\frac{r}{2\sigma^2}\right)^2} \dots$ Fast decaying proximity kernel

$\sigma^2 \dots$ The variance (kernel width)

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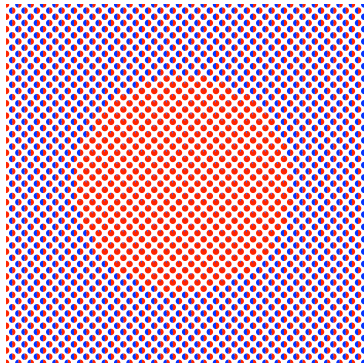


Figure: Toy example of 30×30 grid.

Attraction Potential

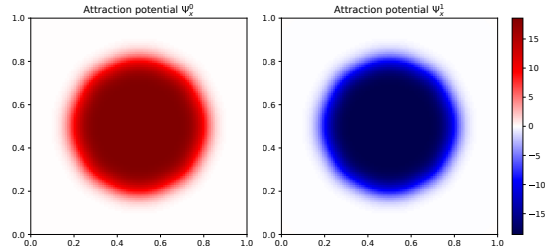
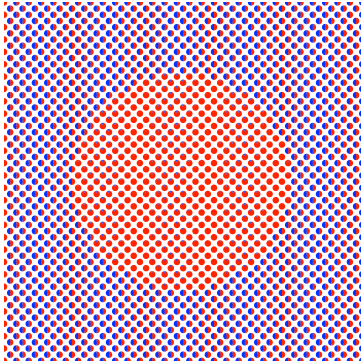


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Attraction Potential

Directions are obtained through partial differentiation of the scalar potential field.

Vector field $\nabla\Phi^k$

$$\Phi^k(p) = \sum_{x \in X} \omega_x^k \theta(\|P(x) - p\|)$$

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$$\nabla\Phi_x^k := \nabla\Phi^k(P(x))$$

$$\nabla(.) = \left\{ \frac{\partial(.)}{\partial x}, \frac{\partial(.)}{\partial y} \right\}$$

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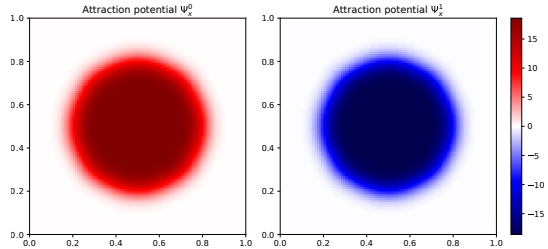
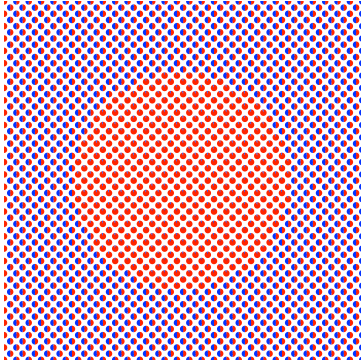


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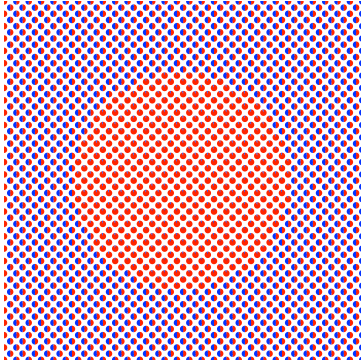
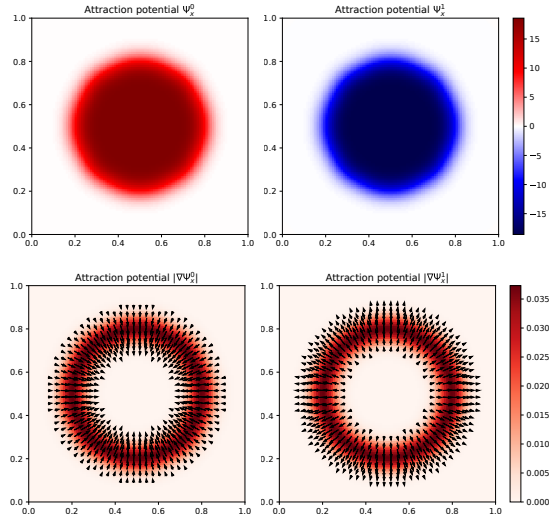


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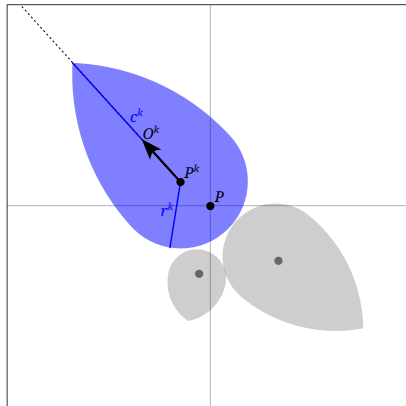
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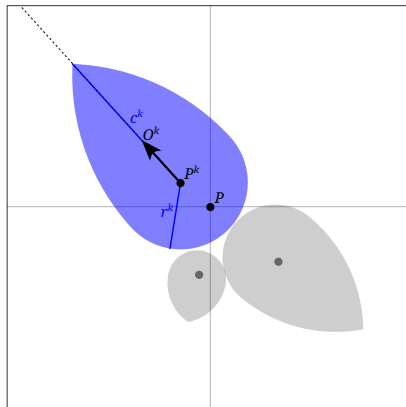
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Marker Layouting

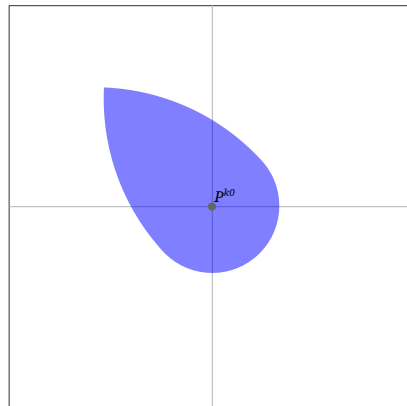
```
LAYOUTING( $x$ )
 $\mathbf{P}_x \leftarrow \{P(x) : k \in K\}$ 
 $\tilde{K} \leftarrow \text{sorted}(\{k \in K : w_x^k > 0\})$ 
if  $|\tilde{K}| > 1$  then
     $k_0 \leftarrow \text{pop}(K)$ ,  $k_1 \leftarrow \text{pop}(K)$ 
     $\mathbf{P}_x[k_1] \leftarrow P(x) + \nabla \hat{\Phi}_x^{k_1}(r^{k_0} + r^{k_1})$ 
    Readjust to center in direction  $-\nabla \Phi_x^{k_1}$ 
    while  $|\tilde{K}| > 0$  do
         $k \leftarrow \text{pop}(K)$ 
        Compute candidate positions  $\tilde{\mathbf{P}}^k$  for  $k$ 
        Discard invalid positions (intersections)
         $\mathbf{P}_x[k] \leftarrow \arg \min_{p \in \tilde{\mathbf{P}}^k} \{\angle(\nabla \Phi^k(x), p)\}$ 
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return  $\mathbf{P}_x$ 
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Marker Layouting

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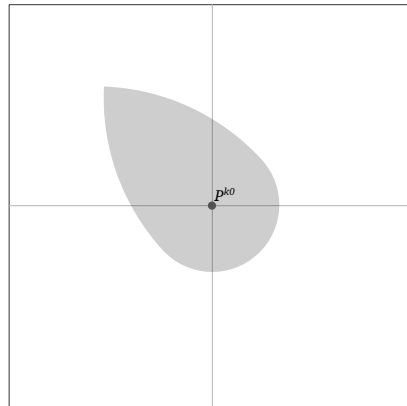


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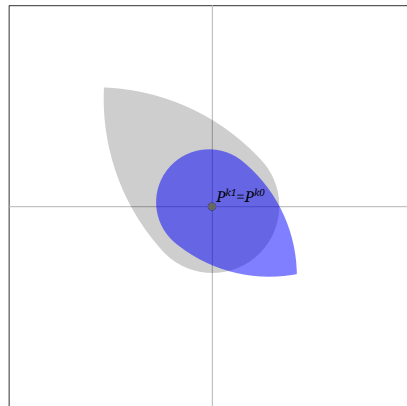


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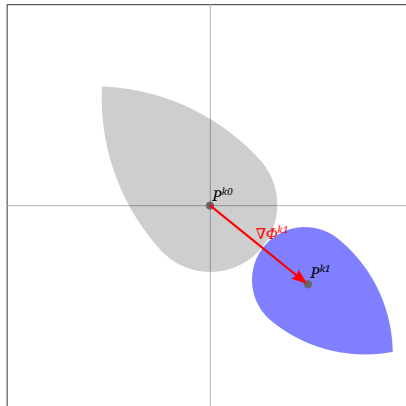


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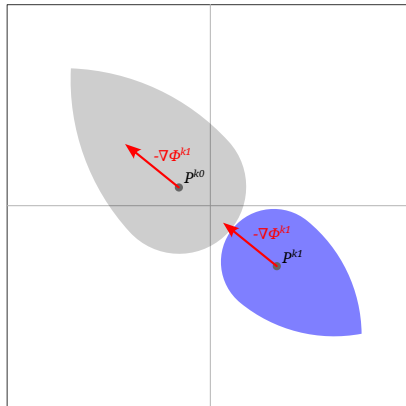


Marker Layouting

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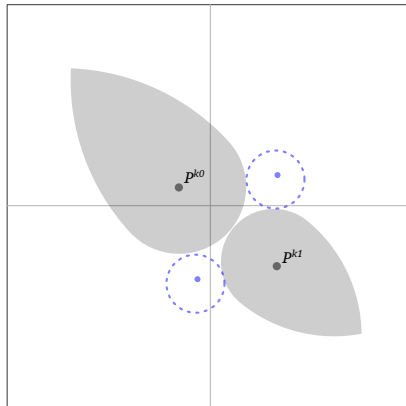


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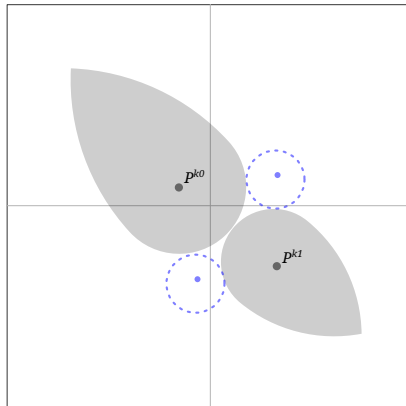


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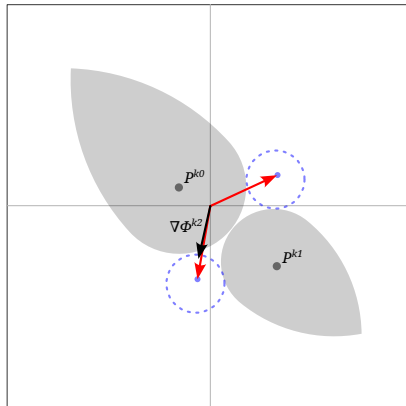


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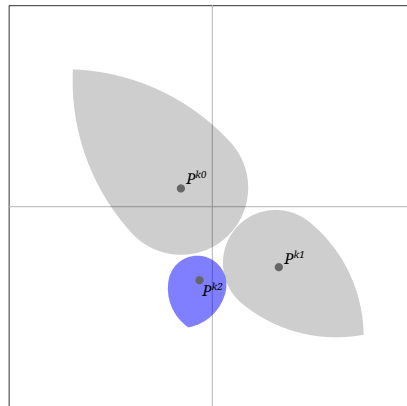


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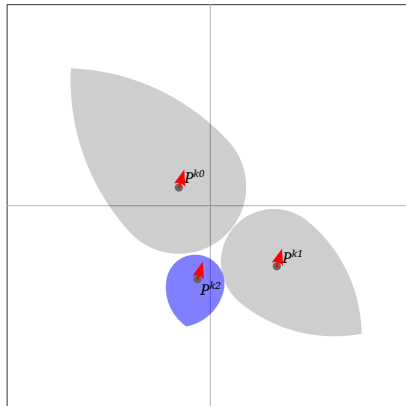


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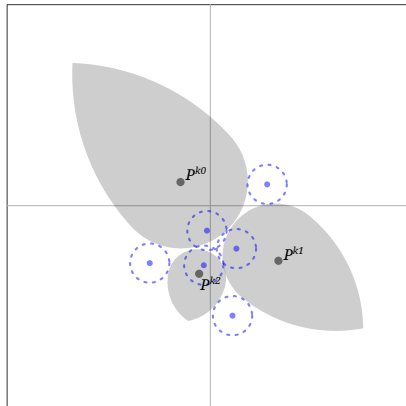


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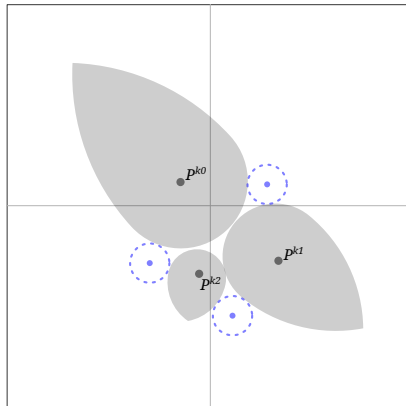


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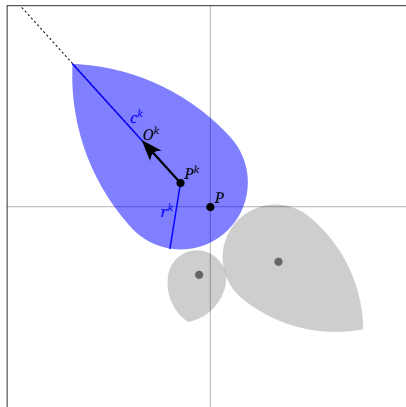
Droplet Marker

A droplet marker consists of a circular base shape, equipped with a protruding tail.

Droplet definition

Droplet $D_x^k = \langle r_x^k, \varepsilon_x^k, O_x^k, P_x^k \rangle$ of class k for pixel x with:

r_x^k	... The radius	$r_x^k \propto w_x^k$ ✓
ε_x^k	... The excentricity	$\varepsilon_x^k = r_x^k / c_x^k, c_x^k \propto \nabla \Phi_x^k $ ✓
O_x^k	... The orientation	$O_x^k \propto \nabla \Phi_x^k$ ✓
P_x^k	... The center point	$P_x^k = P_x^k(\nabla \Phi_x^k, \{P_x^{k'}\}_{k' \in K \setminus \{k\}})$



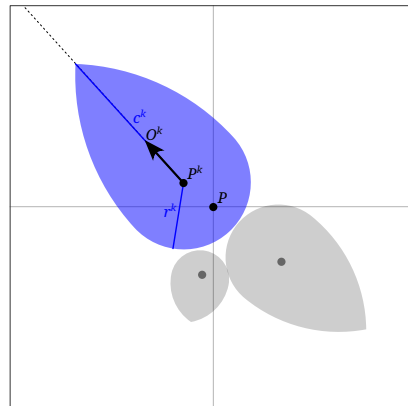
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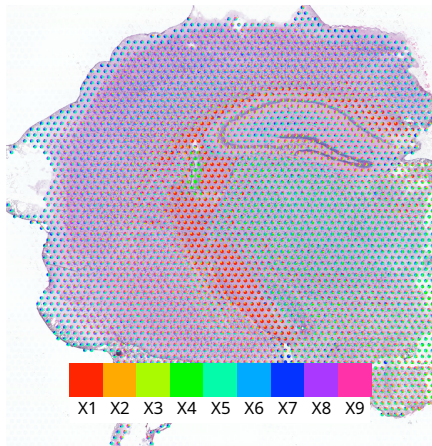
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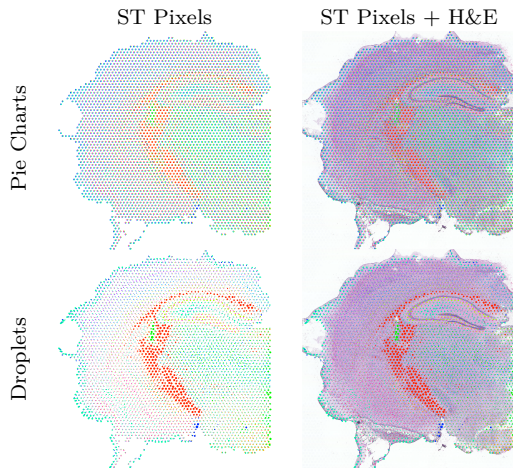


Use Case: Tissue sample with deconvolved cell-types

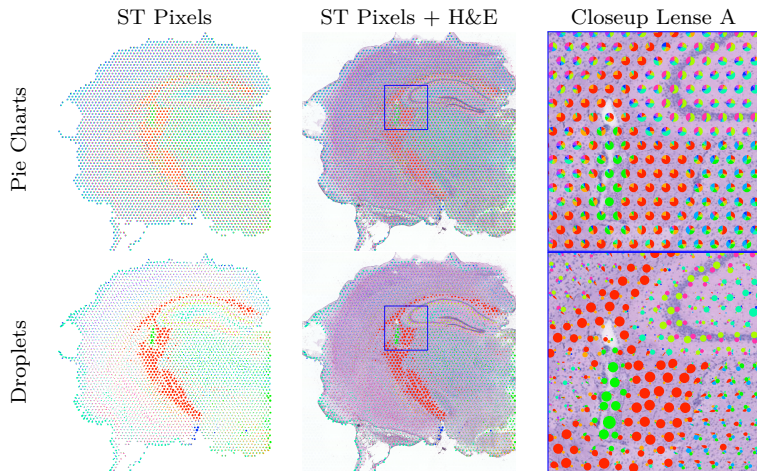


- Regular grid superimposed to H&E stain of tissue sample
- $|X| = 3,499$ pixels
- $|K| = 9$ classes

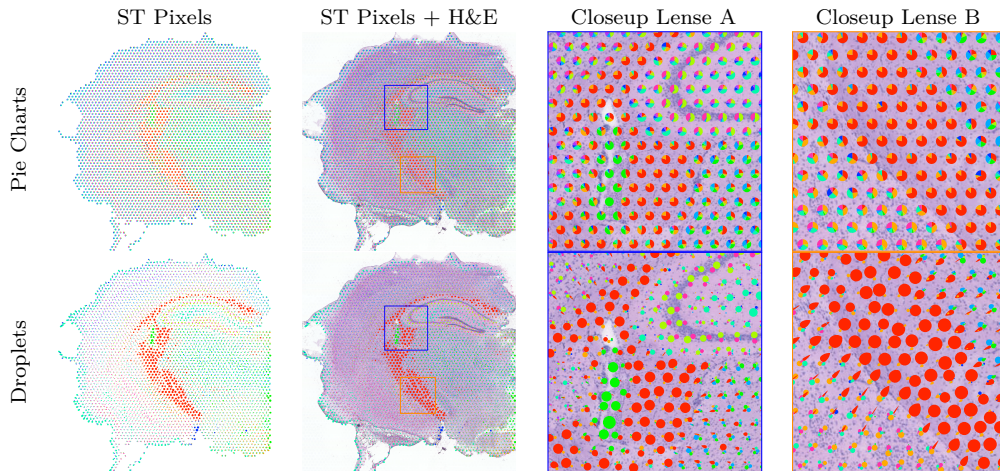
Tissue sample with deconvolved cell-types



Tissue sample with deconvolved cell-types

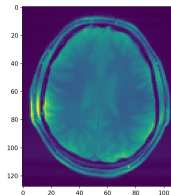


Tissue sample with deconvolved cell-types



Discussion & Outlook

- Additional Use Cases
 - MRI images
 - Sociodemographical data
- Technical improvements
 - Efficiency
 - Non-regular grids
 - Adaptation of the underlying image



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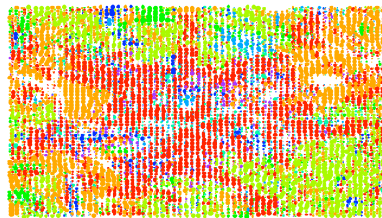
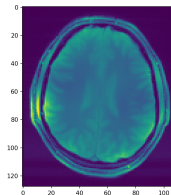


Figure: Berlin

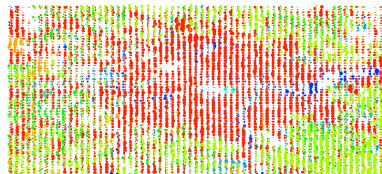


Figure: London

Discussion & Outlook

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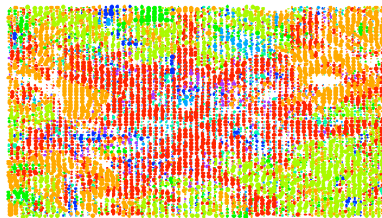
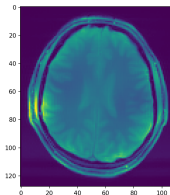


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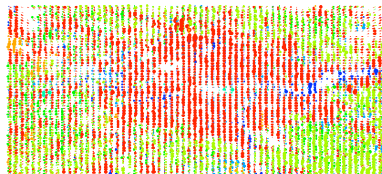
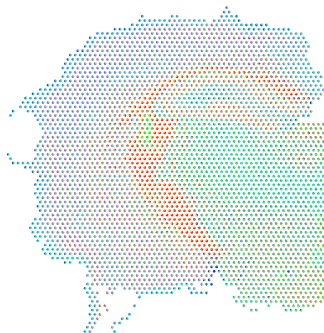
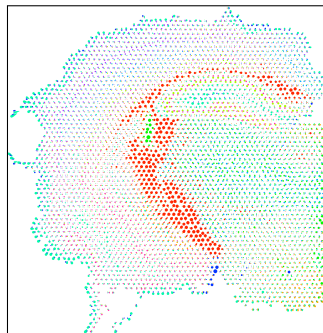


Figure: London

Conclusion



Reference: Pie Charts



Droplets

- Python implementation @ <https://pluto.cgv.tugraz.at/slengauer/droplets>
- Typescript implementation @ <https://pluto.cgv.tugraz.at/hereditary/droplets-interactive>

The research was funded by the EU Horizon project HEREDITARY (<https://hereditary-project.eu>)

Bibliography I

- [1] B. F. Miller, F. Huang, L. Atta, A. Sahoo, and J. Fan, **Reference-free cell type deconvolution of multi-cellular pixel-resolution spatially resolved transcriptomics data**, *Nature communications*, vol. 13, no. 1, p. 2339, 2022.
- [2] E. R. Tufte and P. R. Graves-Morris, **The visual display of quantitative information**. Graphics press Cheshire, CT, 1983, vol. 2.
- [3] E. Mörth, I. S. Haldorsen, S. Bruckner, and N. N. Smit, **Paraglyder: Probe-driven interactive visual analysis for multiparametric medical imaging data**, *Advances in Computer Graphics*, Springer, 2020, pp. 351–363, ISBN: 978-3-030-61864-3. DOI: [10.1007/978-3-030-61864-3_29](https://doi.org/10.1007/978-3-030-61864-3_29).

Bibliography II

- [4] M. Lu, S. Wang, J. Lanir, *et al.*, **Winglets: Visualizing association with uncertainty in multi-class scatterplots**, *IEEE TVCG*, vol. 26, no. 1, pp. 770–779, 2020. DOI: [10.1109/TVCG.2019.2934811](https://doi.org/10.1109/TVCG.2019.2934811).
- [5] **Chapter 3: Gestalt theory**,
<https://chrisbrejon.com/cg-cinematography/chapter-3-gestalt-theory/>,
Accessed: 2025-01-21.

Gestalt principles

- **Continuity:** viewers group elements that seem to follow a continuous path
- **Common Fate:** tendency to group elements which seem to move in a common direction

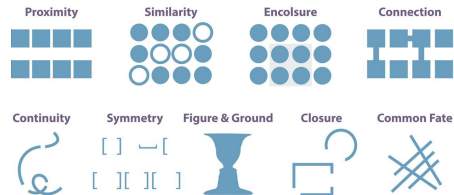


Figure: Gestalt principles [5]

Locally Adaptive Kernel Density

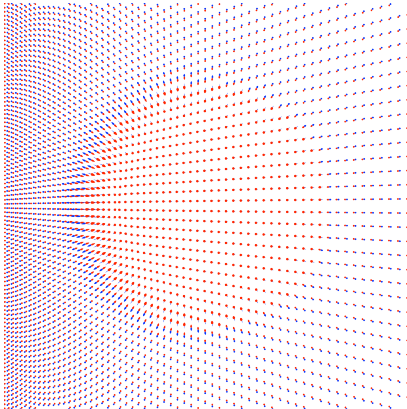


Figure: Static variance σ^2

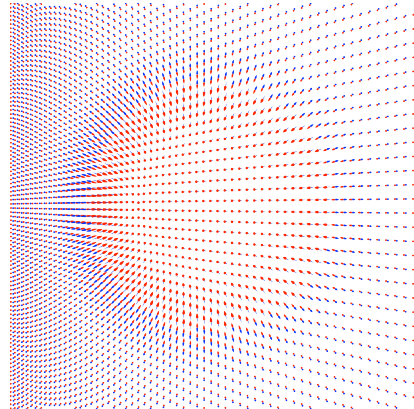


Figure: Locally adaptive variance $\sigma^2 = \sigma^2(p)$